Short Papers

Exact Wave Resistance of Coaxial Regular Polygonal Conductors

RYUITI TERAKADO

Abstract —**There are various cases in which one can evaluate exactly the wave resistance of coaxial conductors such that each of the inner and outer conductors is a regular polygon. They are obtained by conformal mapping using complete elliptic integrals. Such typical examples each with exact wave resistances are shown.**

I. Introduction

Transmission lines formed by regular polygonal coaxial inner and outer conductors whose wave resistances can be determined exactly are useful as a standard of approximation for the wave resistances of various transmission lines [1]–[3]. In this paper, we consider coaxial conductors whose cross section of either the inner or outer conductor is an equilateral and equiangular polygon.

The dielectric medium is taken to be free space.

If the cross section of a regular polygonal coaxial conductor has n mirror symmetric lines, we divide the cross section into symmetrical $2n$ parts. Let $R_\square$ denote the two-dimensional geometrical resistance between the inner and outer conductors of the partial region. Then the wave resistance of the coaxial line can be written

$$R = R_\square R_0 / 2n \tag{1}$$

where $R_0 = 120\pi = 377\ \Omega$.

The symbols of radii used in this paper are as follows:

- r_1 radius of circle circumscribed about an inner conductor ($r_1 = 1$ in all cases),
- r_2 radius of circle inscribed in an outer conductor,
- r_3 radius of circle inscribed in an inner conductor,
- r_4 radius of circle circumscribed about an outer conductor.

II. Equilateral and Equiangular Polygonal Inner Conductor

In Fig. 1, A is the middle point and B is the end point of a side of an inner equilateral and equiangular polygon of a cross section. Let O denote the center of the regular polygon. Points C and D are placed, respectively, on the lines OB and OA. Furthermore, point D is located on the line bisecting $\angle ABC$. Point C is determined by the relation of two right-angled triangles $\Delta BAD \equiv \Delta BCD$. Then the quadrilateral $ABCD$ can be mapped conformally to a semi-circle based on its symmetry. The procedure is as follows. We regard point B of each region, that is, the quadrilateral $ABCD$ and the semi-circle $ABCD$, as a source of lines of electric force, and regard the part ADC of the circumference of each region as a sink. In the semi-circle, any radius coincides then with a line of electric force. In the quadrilateral, however, it is not simple to draw exactly all of the lines of electric force. The three segments BA, BD, and BC coincide with lines of electric force,

Manuscript received April 11, 1984; revised September 17, 1984.

The author is with the Department of Electrical Engineering, Ibaraki University, Hitachi, Ibaraki, 316 Japan.

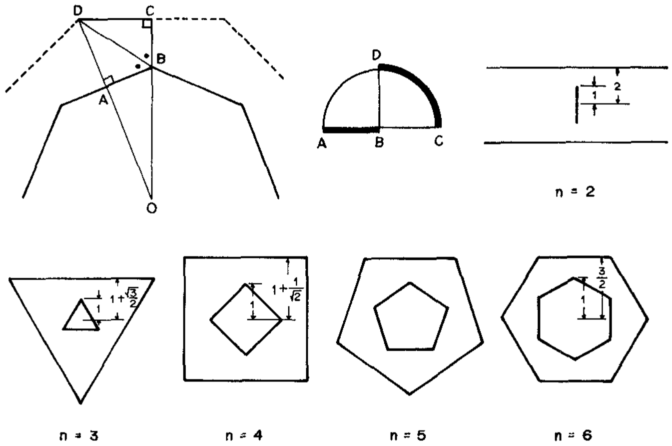

Fig. 1. Coaxial regular polygon cross sections with the modulus of geometrical resistance: $1/\sqrt{2}$.

and it is clear that these lines of electric force correspond to the same lines of the semi-circle. The geometrical resistance $R_\square$ between AB and CD can be obtained by the successive conformal mapping of the semi-circle to a half-plane [4].

In Fig. 1, the modulus of the complete elliptic integrals $K(k)$ and $K'(k)$ for geometrical resistance is $k = 1/\sqrt{2}$, and $R_\square = K'(k)/K(k) = 1$ is clear. Therefore, the wave resistance of the type of Fig. 1 having an equilateral and equiangular polygonal inner conductor with n side is

$$R = R_0 / 2n. \tag{2}$$

Moreover, Fig. 1 shows concrete examples for $n = 2, 3, \cdots, 6$. In this case, an outer polygon becomes necessarily an equilateral and equiangular polygon. Accordingly, taking radius $r_1 = 1$, other radii are expressed as follows:

$$r_2 = 1 + \sin\pi/n \quad r_3 = \cos\pi/n \quad r_4 = \tan(n+2)\pi/4n. \tag{3}$$

In Fig. 2, $\angle ABC$ is divided into three equal angles, and the three right-angled triangles are geometrically equal. The geometrical resistance can be obtained using complete elliptic integrals for modulus $k = \sqrt{3}/2$

$$R_\square = \frac{K'(\sqrt{3}/2)}{K(\sqrt{3}/2)} = 0.7817010. \tag{4}$$

The wave resistance becomes

$$R = 0.7817010 R_0 / 2n. \tag{5}$$

The radii are expressed as follows:

$$r_2 = \frac{\sin^2(n+2)\pi/3n}{\sin(n-1)\pi/3n} \quad (n \leqslant 4) \tag{6a}$$

$$r_2 = \frac{\sin(n+2)\pi/3n}{\sin(n-1)\pi/3n} \quad (n \geqslant 4) \tag{6b}$$

$$r_3 = \cos\pi/n \tag{6c}$$

$$r_4 = 1 + \frac{\sin\pi/n}{\sin(n-1)\pi/3n}. \tag{6d}$$

In this case, an outer polygon is not equiangular except $n = 4$.

0018-9480/85/0002-0143$01.00 ©1985 IEEE

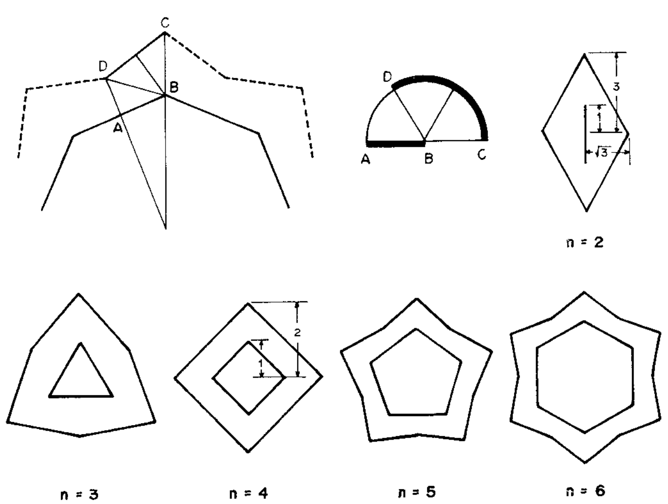

Fig. 2. Coaxial polygon cross sections with inner regular polygon and the modulus of geometrical resistance: $\sqrt{3}/2$.

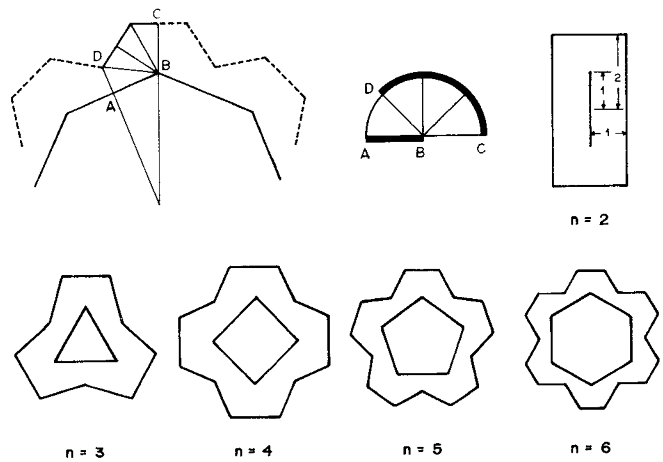

Fig. 3. Coaxial polygon cross sections with inner regular polygon and the modulus of geometrical resistance: $\sqrt{2+\sqrt{2}}/2$.

Similarly, in Fig. 3, having four geometrically equal right-angled triangles, we obtain

$$R_{\square}=\frac{K'\left(\sqrt{2+\sqrt{2}}/2\right)}{K\left(\sqrt{2+\sqrt{2}}/2\right)}=0.6806342 \tag{7}$$

$$r_2=\frac{\sin(3n+6)\pi/8n}{\sin(3n-2)\pi/8n} \tag{8a}$$

$$r_3=\cos\pi/n \tag{8b}$$

$$r_4=\sqrt{1+2\sin\pi/n+\frac{\sin^2\pi/n}{\sin^2(3n-2)\pi/8n}}. \tag{8c}$$

In Fig. 4, the arrangement of three geometrically equal right-angled triangles is different from Fig. 2 having similarly three geometrically equal right-angled triangles. It becomes

$$R_{\square}=\frac{K'(1/2)}{K(1/2)}=1.2792616 \tag{9}$$

$$r_2=(1+2\sin\pi/n)\sin(n+2)\pi/4n \qquad (n\leqslant 6) \tag{10a}$$

$$r_2=\tan(n+2)\pi/4n \qquad (n\geqslant 6) \tag{10b}$$

$$r_3=\cos\pi/n \tag{10c}$$

$$r_4=\tan(n+2)\pi/4n \qquad (n\leqslant 4) \tag{10d}$$

$$r_4=1+2\sin\pi/n \qquad (n\geqslant 4). \tag{10e}$$

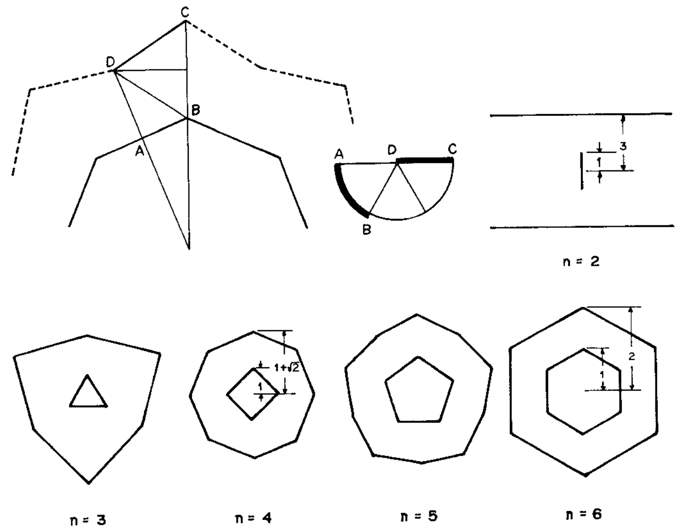

Fig. 4. Coaxial polygon cross sections with inner regular polygon and the modulus of geometrical resistance: 1/2.

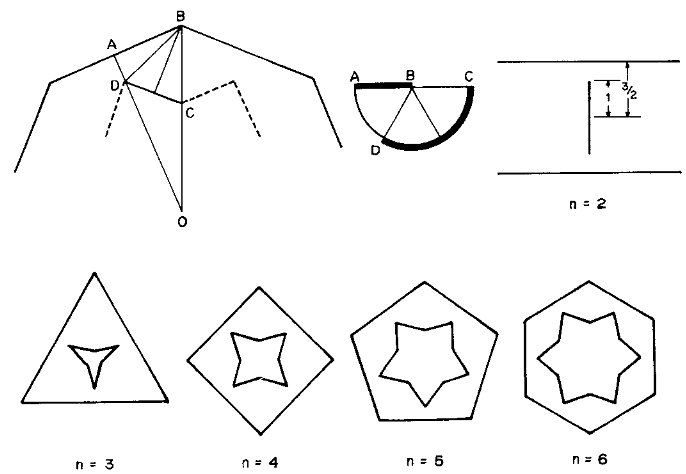

Fig. 5. Coaxial polygon cross sections with outer regular polygon and the modulus of geometrical resistance: $\sqrt{3}/2$.

III. Equilateral and Equiangular Polygonal Outer Conductor

In Fig. 5, points A and B are placed on a side of an outer equilateral and equiangular polygon. Three geometrically equal right-angled triangles are arranged around point B towards the inside. The method deriving the wave resistance is similar to that of an equilateral and equiangular polygonal inner conductor.

The geometrical resistance of the semi-circle in Fig. 5 is equal to that in Fig. 2. Therefore, the wave resistance of Fig. 5 is equivalent to (5). Radii r_2, r_3, and r_4 are as follows:

$$r_2=\frac{1}{2}\left[1+\frac{\sin(2n-4)\pi/3n}{\sin(n-2)\pi/3n}\right] \tag{11a}$$

$$r_3=\frac{\sin(n+1)\pi/3n}{\sin(n-2)\pi/3n}\left[1-\frac{\sin\pi/n}{\sin(2n-2)\pi/3n}\right] \tag{11b}$$

$$r_4=\frac{\sin(n+1)\pi/3n}{\sin(n-2)\pi/3n}. \tag{11c}$$

IV. Applications

Fig. 6 shows different examples. Fig. 6(a) has a regular octagonal inner conductor with eight mirror lines; however, the outer conductor has only four mirror lines. It results from two geomet-

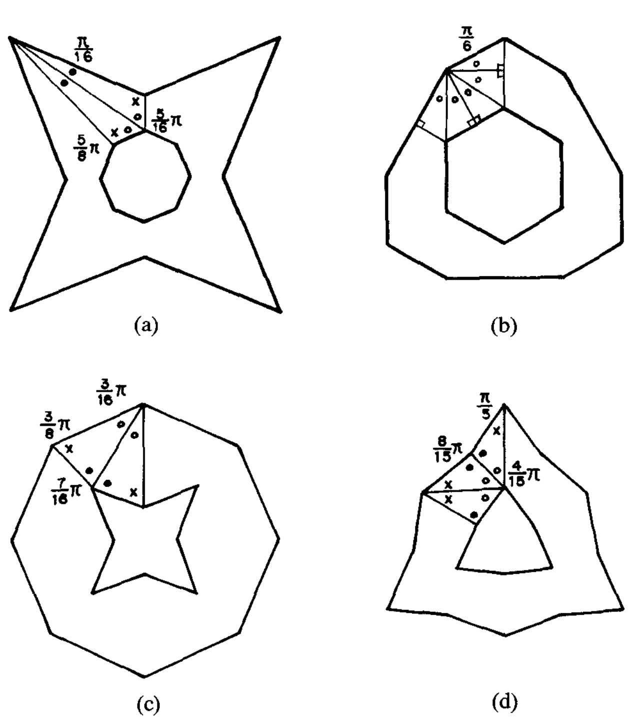

Fig. 6. A family of polygon cross sections suited for the same rule as the preceding figures.

rically equal triangles not arranged on a half side but on a whole side of the inner regular octagon. Furthermore, two geometrically equal triangles are not right-angled. The exact wave resistance becomes (because $R_{\square}=1$)

$$R = R_0/8$$

and $r_2 = 1.765$ and $r_4 = 4.262$.

Fig. 6(b), having five geometrically equal right-angled triangles, is symmetrical with three mirror lines, though the inner regular hexagon has six mirror lines. The exact wave resistance is

$$R = \frac{1}{6}\frac{K'(k)}{K(k)} R_0 = 0.09298 R_0$$

where

$$k = \sqrt{\frac{2(\cos \pi/5 - \cos 3\pi/5)}{(1-\cos 3\pi/5)(1+\cos \pi/5)}} = 0.9713.$$

The radii are $r_2 = 1.5$ and $r_4 = 2.0$.

In Fig. 6(c), two geometrically equal triangles are not right-angled, and they are arranged on a whole side of the outer regular octagon. This case can be considered to be the inversion of Fig. 6(a). Therefore, the exact wave resistance is the same as Fig. 6(a). The radii are $r_2 = 1.631$, $r_3 = 0.4142$, and $r_4 = 1.765$.

In Fig. 6(d), both of the inner and outer conductors are not equiangular polygons. This case is considered as a slight variation for $n = 3$ of Fig. 2 having three geometrically equal right-angled triangles. The wave resistance is equal to Fig. 2 because the procedure of mapping to a semi-circle is similar to Fig. 2. Fig. 6(d) has $R = 0.1303 R_0$, $r_2 = 1.609$, $r_3 = 0.5877$, and $r_4 = 2.433$.

V. Conclusion

Some shapes of coaxial inner and outer regular polygonal conductors can be exactly evaluated on their wave resistance.

The following special cases of this paper coincide with Wheeler [1]:

Herein		*Wheeler*
Fig. 1	$n = 2$	Fig. 23
	$n = 3$	25
	$n = 4$	25
Fig. 2	$n = 2$	26
	$n = 4$	26
Fig. 3	$n = 2$	26.

References

[1] H. A. Wheeler, "Transmission-line conductors of various cross sections," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 73–83, Feb. 1980.

[2] H. J. Riblet, "An accurate determination of the characteristic impedance of the coaxial system consisting of a square concentric with a circle," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 714–715, Aug. 1975.

[3] H. J. Riblet, "An expansion of the Terakado solution with an application," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 2036–2039, Nov. 1982.

[4] R. Terakado, "The characteristic impedance of rectangular coaxial line with ratio 2:1 of outer-to-inner conductor side length," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 124–125, Feb. 1976.

Analysis of the Transmission Characteristics of Inhomogeneous Grounded Finlines

ADALBERT BEYER

***Abstract*—This paper describes a concept for an efficient design of finline tapers that is especially useful in cases when certain quantities have been prescribed with respect to reflection loss and bandwidth. Since abrupt discontinuities are neglected, the analysis is applicable to smooth finline tapers only.**

Various contour functions are investigated for the taper optimization. Experimental results for optimized tapers confirm the design theory.

I. Introduction

Smooth inhomogeneous finlines have already been used as broad-band components like transformers, attenuators, detectors, mixers, and nonreciprocal elements [1], [2], [5].

In the beginning, the design of inhomogeneous finlines was mainly done experimentally, i.e., the cross sections of these tapers had been designed with a general parabolical dependence of the slot widths ($2s$) on the length coordinate (z) with the exponent of the parabola having been determined experimentally [5].

Recently, there have been publications of nonexperimental design procedures, e.g., in [6], where the use of a spectral-domain approach has been suggested. Another concept [9], which considers inhomogeneous finlines that consist of an infinite number of elementary homogeneous finline sections, has proved the realizability of this line.

This paper follows the method presented in [4], which has the considerable advantage of taking into account the influence of the thickness of the metallization and of the longitudinal slits in the housing.

Manuscript received May 2, 1984; revised September 4, 1984.

The author is with Fachgebiet Allgemeine und Theoretische Elektrotechnik, Department of Electrical Engineering, Duisburg University, Bismarckstrasse 81, D-4100 Duisburg 1, Federal Republic of Germany.

0018-9480/85/0002-0145$01.00 ©1985 IEEE